Algebra and Calculus: Homework 10 Solutions

Section 4.5: 6, 22, 28, 42, 44, 52, 62, 66

Q6: Find the solution to the equation $10^{2x-3} = \frac{1}{10}$

$$\begin{aligned} 10^{2x-3} &= 10^{-1} \\ \implies 2x - 3 &= -1 \\ \implies 2x &= 2 \\ \implies x &= 1 \end{aligned}$$

Q22: Solve for x: $e^{3-5x} = 16$, then find an approximation to the solution rounded to 6 decimal places.

$$\begin{aligned} e^{3-5x} &= 16 \\ 3 - 5x &= \ln(16) \\ x &= \frac{1}{5}(3 - \ln(16)) \\ &\approx 0.045482 \end{aligned}$$

Q28: Solve for x: $2(5 + 3^{x+1}) = 100$, then find an approximation to the solution rounded to 6 decimal places.

$$\begin{aligned} 2(5 + 3^{x+1}) &= 100 \\ 5 + 3^{x+1} &= 50 \\ 3^{x+1} &= 45 \\ x + 1 &= \log_3(45) \\ x &= \log_3(45) - 1 = \frac{\ln(45)}{\ln(3)} - 1 \\ &\approx 2.464974 \end{aligned}$$

We used the change of base formula to compute this (your graphing calculator should have a natural log function on it; one could also use the base 10 log).

Q42: Solve the equation $3^{4x} - 3^{2x} - 6 = 0$.

$$\begin{aligned} b &= 3^{2x} \\ b^2 - b - 6 &= 0 \\ (b-3)(b+2) &= 0 \\ \implies b &= \{3, -2\} \end{aligned}$$

But we must eliminate the negative root, as b must be a positive value (it is an exponential). Thus

$$3^{2x} = 3 = 3^1$$
$$\implies 2x = 1$$
$$\implies x = \frac{1}{2}$$

Q44: Solve: $e^x + 15e^{-x} - 8 = 0$.

Let $b = e^x$:

$$b + 15b^{-1} - 8 = 0$$

Multiply both sides by b. We can do this because $b \neq 0$ for any x, so multiplying by b will not cause any problems:

$$b^2 + 15 - 8b = 0$$
$$(b-3)(b-5) = 0$$
$$\implies b = \{3, 5\}$$

We see that both roots are valid. We can substitute e^x back in:

$$e^x = 3 \implies x = \ln(3)$$
$$e^x = 5 \implies x = \ln(5)$$

Q52: Solve for x: $\ln\left(x - \frac{1}{2}\right) + \ln(2) = 2\ln(x)$

$$\ln\left(x - \frac{1}{2}\right) + \ln(2) = 2\ln(x)$$
$$\ln\left([2]\left[x - \frac{1}{2}\right]\right) = \ln\left(x^2\right)$$
$$2x - 1 = x^2$$
$$x^2 - 2x + 1 = 0$$
$$(x-1)^2 = 0$$
$$\implies x = 1$$

Q62: Solve for x: $\log_2\left(x^2 - x - 2\right) = 2$

$$\log_2\left(x^2 - x - 2\right) = 2$$
$$x^2 - x - 2 = 2^2 = 4$$
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$\implies x = \{-2, 3\}$$

Q66: Solve for x: $\ln(x-1)+\ln(x+2)=1$

$$\begin{aligned}
\ln(x-1)+\ln(x+2) &= 1 \\
\ln([x-1][x+2]) &= \ln(e) \\
(x-1)(x+2) &= e \\
x^2+x-2 &= e \\
x^2+x-(2+e) &= 0 \\
x &= \frac{-1\pm\sqrt{1+4(2+e)}}{2} \\
&= \frac{-1\pm\sqrt{4e+9}}{2}
\end{aligned}$$

But we must eliminate the negative root, since otherwise $\ln(x-1)$ will have to take an input that is not in the domain of the function (i.e. for $\ln(y)$ we need $y>0$). Thus,

$$\begin{aligned}
x &= \frac{-1+\sqrt{4e+9}}{2} \\
&\approx 1.728964
\end{aligned}$$