Algebra and Calculus Worksheet: Recitation 1 (9-14-15)

Name: ____________________________

This worksheet is a mix of problems from Prof. Wykes' list of suggested exercises in Homework 1 and 2.

1. **Section 1.3: Question 75**

   Perform the indicated operations and simplify:

$$(3x+2)^2 + 8(3x+2) + 12$$

   Solution:

$$\begin{aligned}(3x+2)^2 + 8(3x+2) + 12 &= (3x+2)(3x+2+8) + 12 \\ &= (3x+2)(3x+10) + 12 \\ &= 9x^2 + 30x + 6x + 20 + 12 \text{ (FOIL)} \\ &= 9x^2 + 36x + 32\end{aligned}$$

2. **Section 1.3: Question 91**

   Factor the expression completely. Begin by factoring out the lowest power of each common factor.

$$x^{\frac{5}{2}} - x^{\frac{1}{2}}$$

   Solution:

$$\begin{aligned}x^{\frac{5}{2}} - x^{\frac{1}{2}} &= x^{\frac{1}{2}}\left(x^{\frac{4}{2}} - 1\right) \\ &= x^{\frac{1}{2}}\left(x^2 - 1\right) \\ &= x^{\frac{1}{2}}(x-1)(x+1)\end{aligned}$$

3. **Section 1.3: Question 127**

   Factor the expression completely:

$$5(x^2+4)^4(2x)(x-2)^4 + (x^2+4)^5(4)(x-2)^3$$

   Solution:

$$\begin{aligned}5(x^2+4)^4(2x)(x-2)^4 + (x^2+4)^5(4)(x-2)^3 &= (x^2+4)^4(x-2)^3\left[5(2x)(x-2) + (x^2+4)(4)\right] \\ &= (x^2+4)^4(x-2)^3\left[10(x^2-2x) + 4x^2 + 16\right] \\ &= (x^2+4)^4(x-2)^3\left[10x^2 - 20x + 4x^2 + 16\right] \\ &= (x^2+4)^4(x-2)^3\left[14x^2 - 20x + 16\right] \\ &= 2(x^2+4)^4(x-2)^3\left[7x^2 - 10x + 8\right]\end{aligned}$$

4. **Section 1.4: Question 27**

   Perform the necessary operations to simplify the expression:

$$\frac{x^2+2x-15}{x^2-25}\cdot\frac{x-5}{x+2}$$

Solution:

$$\begin{aligned}\frac{x^2+2x-15}{x^2-25}\cdot\frac{x-5}{x+2} &= \frac{(x+5)(x-3)}{(x+5)(x-5)}\cdot\frac{x-5}{x+2}\\ &= \frac{\cancel{(x+5)}(x-3)}{\cancel{(x+5)}\cancel{(x-5)}}\cdot\frac{\cancel{x-5}}{x+2}\\ &= \frac{x-3}{x+2}\end{aligned}$$

5. **Section 1.4: Question 35**

   Perform the necessary operations to simplify the expression:

$$\frac{\frac{x^3}{x+1}}{\frac{x}{x^2+2x+1}}$$

Solution:

$$\begin{aligned}\frac{\frac{x^3}{x+1}}{\frac{x}{x^2+2x+1}} &= \frac{\frac{x^3}{x+1}}{\frac{x}{(x+1)^2}}\\ &= \frac{\frac{x^3}{\cancel{x+1}}(x+1)^{\cancel{2}}}{\frac{x}{\cancel{(x+1)^2}}\cancel{(x+1)^2}}\\ &= \frac{x^{\cancel{3}2}(x+1)}{\cancel{x}}\\ &= x^2(x+1)\end{aligned}$$

6. **Section 1.4: Question 75**

   Simplify the fractional expression:

$$\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$$

Solution:

$$\begin{aligned}
\frac{\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}}{h} &= \frac{\left(\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}\right) x^2(x+h)^2}{x^2(x+h)^2 h} \\
&= \frac{x^2 - (x+h)^2}{x^2(x+h)^2 h} \\
&= \frac{x^2 - (x^2 + 2xh + h^2)}{x^2(x+h)^2 h} \\
&= \frac{-2xh - h^2}{x^2(x+h)^2 h} \\
&= -\frac{\cancel{h}(2x+h)}{x^2(x+h)^2 \cancel{h}} \\
&= -\frac{2x+h}{x^2(x+h)^2}
\end{aligned}$$

7. **Section 1.5: Question 21**

   Solve the equation (and simplify):

$$2(1-x) = 3(1+2x) + 5$$

   Solution:

$$\begin{aligned}
2(1-x) &= 3(1+2x) + 5 \\
2 - 2x &= 3 + 6x + 5 \\
-2x - 6x &= 3 + 5 - 2 \\
-8x &= 6 \\
x &= -\frac{6}{8} = -\frac{3}{4}
\end{aligned}$$

8. **Section 1.5: Question 47**

   Find all real solutions of the equation by factoring.

$$x^2 - 7x + 12 = 0$$

   Solution:

$$\begin{aligned}
x^2 - 7x + 12 &= 0 \\
(x-4)(x-3) &= 0 \\
x - 4 &= 0 \implies x = 4 \\
x - 3 &= 0 \implies x = 3 \\
x &= \{3, 4\}
\end{aligned}$$

9. **Section 1.5: Question 123**

   Solve for x (a is a constant...just like any other constant!):

$$x^4 - 5ax^2 + 4a^2 = 0$$

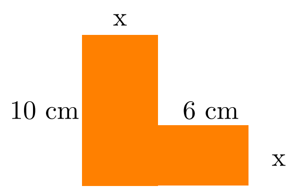
Solution:

$$x^4 - 5ax^2 + 4a^2 = 0$$
$$(x^2 - a)(x^2 - 4a) = 0$$
$$x^2 - a = 0 \implies x = \pm\sqrt{a}$$
$$x^2 - 4a = 0 \implies x = \pm 2\sqrt{a}$$
$$x = \{\pm\sqrt{a}, \pm 2\sqrt{a}\}$$

10. **Section 1.7: Question 47 (part (a))**

Find the length x in the figure. The area of the shaded region is given (do **not** assume anything is drawn to scale).

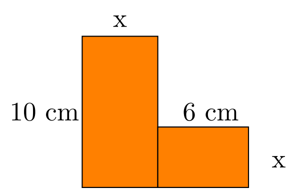
(a)



Area = 144 cm^2

Solution

It is best to decompose this L-shape into two rectangles:



Area = 144 cm^2

The rectangle on the left has sides of length 10 cm and x, and the rectangle on the right has sides of length 6 cm and x. So we have

$$A_{\text{left}} = 10x$$
$$A_{\text{right}} = 6x$$
$$A = A_{\text{left}} + A_{\text{right}} = 144\text{cm}^2$$
$$16x = 144$$
$$x = 9$$

So the missing side is 9 cm long.

11. **Section 1.7: Question 69**

Distance, Speed, and Time: Wendy took a trip from Davenport to Omaha, a distance of 300 mi. She traveled part of the way by bus, which arrived at the train station just in time for Wendy to complete her journey by train. The bus averaged 40 mi/h, and the train averaged 60 mi/h. The entire trip took $5\,\frac{1}{2}$ h. How long did Wendy spend on the train?

Solution: Wendy arrived *just in time* for her to complete her journey by train, which implies that the waiting time in between taking the bus and taking the train is negligible. Thus, assume that the total time Wendy spent traveling = time on bus + time on train.

There are two vehicles whose times we need to know (i.e. if we know the time spent on the bus, we can compute the time spent on the train). Thus, we expect to have *two* equations for *two* times (call them t_{bus} and t_{train}):

We have already stated the first equation, that the total time spent traveling is the sum of the individual times:

$$t_{bus} + t_{train} = 5\frac{1}{2} \text{ (leaving out units here)}$$

Next we can use the following formula:

$$d = vt$$

where d is the distance traveled, v is the average velocity and t is the time spent traveling. We know the velocity traveled by each vehicle ($v_{bus} = 40, v_{train} = 60$), so we can represent each individual distance using the formula:

$$\begin{aligned} d_{bus} &= 40t_{bus} \\ d_{train} &= 60t_{train} \end{aligned}$$

And the sum of these distances equals the total distance:

$$\begin{aligned} d_{bus} + d_{train} &= d \\ 40t_{bus} + 60t_{train} &= 300 \end{aligned}$$

So our two equations are:

$$\begin{aligned} t_{bus} + t_{train} &= 5\frac{1}{2} \\ 40t_{bus} + 60t_{train} &= 300 \end{aligned}$$

Here I solve via substitution:

$$\begin{aligned} t_{bus} &= 5\frac{1}{2} - t_{train} \\ 40\left(5\frac{1}{2} - t_{train}\right) + 60t_{train} &= 300 \\ (220 - 40t_{train}) + 60t_{train} &= 300 \\ 20t_{train} &= 80 \\ t_{train} &= 4 \text{ hours} \\ \implies t_{bus} &= 5\frac{1}{2} - 4 = 1\frac{1}{2} \text{ hours} \end{aligned}$$