

Calc I: Worksheet (Week 11)

Name: _____

Derivatives and the Shapes of Graphs

1. Find the local maximum and minimum values of using both the First and Second Derivative Tests. Which method do you prefer?

(a) $f(x) = 1 + 3x^2 - 2x^3$

(b) $f(x) = \frac{x^2}{x-1}$

2. For each of the following:

- Find the intervals on which f is increasing or decreasing.
- Find the local maximum and minimum values of f .
- Find the intervals of concavity and the inflection points.

(a) $f(x) = 2x^3 + 3x^2 - 36x$

(b) $f(x) = \frac{x}{x^2+1}$

(c) $f(x) = \cos^2 x - 2 \sin x, 0 \leq x \leq 2\pi$

3. Look at the figure below:

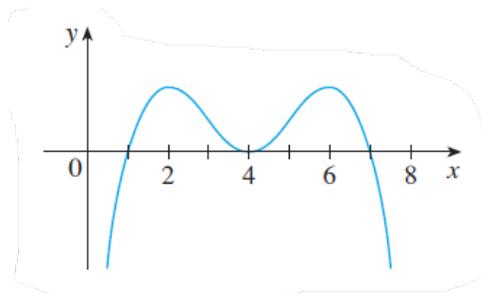


Figure 1:

In each part state the x -coordinates of the inflection points of f . Give reasons for your answers.

- (a) The curve is the graph of f .
(b) The curve is the graph of f' .
(c) The curve is the graph of f'' .

4. Sketch the graph of a function such that $f'(x)$ and $f''(x)$ are always negative.

Curve Sketching

5. Sketch the following curves:

(a) $y = \frac{x}{x-1}$

(b) $y = x\sqrt{5-x}$

(c) $y = (4-x^2)^5$

(d) $y = \frac{\sin x}{1+\cos x}$

(e) $y = \frac{e^x}{x^2}$

(f) $y = xe^{-\frac{1}{x}}$

Optimization Problems

6. Find two numbers whose difference is 100 and whose product is a minimum.
7. What is the minimum vertical difference between the parabolas $y = x^2 + 1$ and $y = x - x^2$?
8. Find the dimensions of a rectangle with perimeter 100 m whose area is as large as possible.
9. Find points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$.
10. A cone with height h is inscribed in a larger cone with height H so that its vertex is at the center of the base of the larger cone. Show that the inner cone has maximum volume when $h = \frac{H}{3}$.
11. The manager of a 100-unit apartment complex knows from experience that all units will be occupied if the rent is \$800 per month. A market survey suggests that, on average, one additional unit will remain vacant for each \$10 increase in rent. What rent should the manager charge to maximize revenue?
12. A baseball team plays in a stadium that holds 55,000 spectators. With ticket prices at \$10, the average attendance had been 27,000. When ticket prices were lowered to \$8, the average attendance rose to 33,000.
 - (a) Find the demand function, assuming that it is linear.
 - (b) How should ticket prices be set to maximize revenue?