

Math-UA 121: Worksheet Two

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Section 1.5 - Continuity

1. Use the Intermediate Value Theorem to show that the equation had at least one real root.

(a) $\cot x = x$

(b) $x^4 + x - 3 = 0$

(c) $\sqrt{5}x = 1 - x$.

2. (a) For what values of a and b is the following function continuous on $(-\infty, \infty)$. $f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & x < 3 \\ ax^2 - bx + 3, & 3 < x < 4 \\ 2x - 2a + b, & x \geq 4 \end{cases}$

(b) Find the value of c (if it exists) that make f continuous everywhere. $f(x) = \begin{cases} cx + 1, & x \leq 3 \\ cx^2 - 1, & x > 3 \end{cases}$

(c) Suppose $f(x) = \begin{cases} \sin(x), & x \leq c \\ a + bx, & x > c \end{cases}$ where a, b, c are constants. If c and b are given, find all values of a such that f is continuous at the point $x = c$.

Section 1.4 - Limits Involving Infinity

Evaluate the limit, if it exists.

3. $\lim_{x \rightarrow \infty} \cos\left(\frac{1}{x}\right)$

7. $\lim_{x \rightarrow -\infty} \frac{3x^3 + 4x - 7}{x^3}$

4. (a) $\lim_{x \rightarrow 2^-} \frac{x^2 + x}{x - 2}$

8. $\lim_{x \rightarrow \infty} \frac{\cos^2(x)}{x^2}$

(b) $\lim_{x \rightarrow 2^+} \frac{x^2 + x}{x - 2}$

9. $\lim_{x \rightarrow -\frac{\pi}{2}} \cot(x)$

(c) $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$

10. (a) $\lim_{x \rightarrow a} \sqrt{x^2}$, where $a < 0$

(b) $\lim_{x \rightarrow a} \sqrt{x^2}$, where $a > 0$

5. $\lim_{t \rightarrow \infty} \frac{\sqrt{t-4}}{t+47}$

6. $\lim_{x \rightarrow \infty} \frac{x^2 + 2x - 7}{x^2 + 9}$

11. $\lim_{x \rightarrow 5^-} \frac{x-4}{\sqrt{4x^2+5}}$

Find the horizontal and vertical asymptotes of the function, if they exist.

12. $f(x) = x + 4$

16. $f(x) = \frac{x^3 + 3x + 2}{x^3 + 27}$

13. $f(x) = \frac{x-2}{x^3+1}$

17. $f(x) = \frac{x^7 + x^5 + 90x^2 + 86}{x^6 + 4x^3 + 2x^3 + x}$

14. $f(x) = \frac{x-67}{x+57}$

18. $f(x) = \frac{x^3 + 2x + 7}{7x^6 + 2x + 9}$

15. $f(x) = \frac{x^2 - 2x + 7}{\sqrt{x^5 + 7x - 2}}$

19. $f(x) = \frac{x-4}{\sqrt{2x^2 + 5x + 2}}$